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Field theoretic treatment of Ionization of H-atom by protons

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In this paper attempt is made to obtain the effect of statistics on the ionization crossection of hydrogen like atoms by the proton impact. An exchange term is obtained, whose evaluation is really difficult. Its effect has been estimated to show a reduction in the value of the differential crosssection.

Introduction

The problem of ionization of hydrogenlike atoms by proton impact has been of considerable interest from a long time. Mott & Massey (1949) have treated it with the help of usual non-relativistic Born approximation. And almost the same line of thinking has been followed by Bates & Griffing (1993) in re-evaluation of the same. But an important feature namely, the identity of the two protons has been overlooked in both the treatments. In the computations mentioned above there is no way to take into account this identity, not even with the help of usual antisymmetrization technique. This is where statistics comes into play. We have tried to show the effect of this identity on the cross-section for the above mentioned process. Quantum field theory takes care of this identity, with the aid of the anticommutation rules for the field operators for the protons.

Formulation of the problem

The initial and final state vectors and the interaction Hamiltonian of the whole system is written (Roy 1960) as

$$H' = \bar{\psi}\,\gamma_\mu\, A_\mu\,(x)\,\psi\,(x) + \bar{\phi}\,(x')\,\gamma_\nu\; A_\nu\,(x')\,\phi\,(x') \tag{1}$$

$$|\,\psi_i> = \int dk dk'\, g(k,k')\, a^+_k\,(k)\, A^+\,(k')\, A^+\,(p_1)\,|\,0>$$

and

$$|\,\psi_f> = a^+\,(p_f)\, A^+\,(p_1')\, A^+\,(p_2')\,|\,0> \tag{2}$$

where ψ is the electron field operator and ϕ is the same for proton. $g(k,k')$ being the fourier transform of the bound state wave function. The matrix element of the process is given by :

$$M_{fi} = \frac{e^2}{2!} < \psi_f \,|\, \int\int H'(x)\, H'(x') dx dx' \,|\, \psi_i > \tag{3}$$

Equation (3) with the help of (1) & (2) gives the following expression for M_{fi},

$$M_{fi} = M_{f_1}(PP) + M_{f_1}(Pe) \tag{4}$$

where,

$$\text{a)}\quad M_{fi}(Pe) = \left[1 + \frac{(p_1 - p_2')^2}{Mmc^2}\right] \frac{g(p_f + p_2' - p, p_1')}{(p_1 - p_2')^2}$$

$$-\left[1 + \frac{(p_1 - p_1')^2}{Mmc^2}\right] \times g(p_f + p_1' - p_1, p_2') \times \frac{1}{(p_1 - p_1')^2} \tag{5}$$

coming from proton-electron interaction term.

$$\text{b)}\quad M_f(PP) = \left[1 + \frac{(p_1 - p_2)^2}{M^2c^2}\right] \frac{g(p_f, p_1' + p_2' - p_1)}{(p_1 - p_2')^2}$$

$$-\left[1 + \frac{(p_1 - p_1')^2}{M^2c^2}\right] \frac{g(p_f, p_1' + p_2' - p_1)}{(p_1 - p_1')^2} \tag{6}$$

which is due to proton-proton term.

In these equations P_1', P_2' are the outgoing momenta of the two protons and P_1 that of incoming proton. These are the relativistic generalisation of those of Bates & Griffing (1963). It can be easily seen that equation (6) is zero in non-relativistic limit due to orthogonality of the wave functions.

Transformation to Centre of Mass System

Due to the energy range covered by the experiments (upto 50 Kev) we usually deal with the NR limit of equation (5). In NR limit it reduces to the familiar expression of Mott & Massey (1949), with the exception of the exchange term. When transformed to cm system first term becomes,

$$\frac{1}{4\pi}\int d\bar{x}\, d\bar{r} \exp\left(\frac{-iXm}{\hbar}(\bar{v}_1' - \bar{v}_f) + \frac{i\bar{r}M}{\hbar}(v_q - v_q)\right) \frac{\phi(X)}{|\bar{r} - \bar{X}|} \tag{7}$$

where $\bar{v}_1, \bar{v}_2$ etc. are velocities corresponding to moments P_1, P_2. $\bar{v}_p, \bar{v}_q$ being the relative velocities corresponding to the initial and final state of the system. Equation (7) is then seen to be equal to

$$\frac{1}{(\vec{K}_q - \vec{K}_q)^2\{1 + a_0^2(\vec{K}_p - \vec{K}_q + \vec{K}_1)^2\}^2} \tag{8}$$

when $\phi(X)$ is taken to be the 1S-state, while the exchange term

reduces to

$$\frac{1}{(\vec{K}_p + \vec{K}_q + \vec{K}_1)^2 \{1 + a_0^2(\vec{K}_p + \vec{K}_p)^2\}^2}$$

in which $\vec{K}_p$ and $\vec{K}_q$ are defined by

$$\vec{K}_p = \frac{2\pi M_p}{h}, \vec{K}_q = \frac{2\pi M \vec{v}_q}{h} \qquad (9)$$

EVALUATION OF CROSSECTION

For the ejected electron to be in solid angle $d\sigma$, the proton in $d\omega$ and electron momenta within K_1 and $K_1 + dK_1$, the crosssection of the process is then given by,

$$Id\sigma d\omega dK_1 = \frac{4\pi^2 M^2}{h^4} \cdot \frac{K_q}{K_p} \cdot |M_{fi}|^2 d\sigma d\omega dK_1 \qquad ...(10)$$

which with the above expression for M_{fi} gives, after adjustment of proper normalising factors,

$$Id\sigma d\omega dK_1 = \{f(K_p, K_q, K_1) d\sigma d\omega dK_1\} \pi a_0^2 \qquad ...(11)$$

where,

$$f(K_p, K_q, K_1) = \frac{2^{12}\pi^4 M^2 e^4}{h^4} a_0 \cdot \frac{K_q}{K_p} K_1^2 \times$$

$$\left[\frac{1}{(K_p - K_q)^4 \{1 + a_0^2(K_p - K_p + K_1)^2\}^4} + \frac{1}{(K_p + K_q + K_1)^4 \{1 + a_0^2(K_p + K_q)^2\}^4}\right.$$

$$\left. - \frac{2}{(K_p - K_q)^2 (K_p + K_q + K_1)^2 \{1 + a_0(K_p + K_q)^2\}^2 \{1 + aK^2(K_p - K_q + K_1)^2\}^2}\right] \qquad .. (12)$$

First term in equation (12) is the result of usual non-relativistic calculation, while the 2nd and 3rd terms are the results of the exchange effect due to statistics.

For the total crosssection we are to integrate over the angular distributions of the ejected electrons and protons. First term in equation (12) gives

$$\theta_{tot} = \pi a_0^2 \left[\frac{29\pi^4}{\hbar^2 v_p^2}\right]^2 \int_0^{\chi_{max}} \int_{t_1}^{t_2} \frac{d\omega d\chi \chi^2 dt}{t^3 \{1 + (\chi + t)^2\}^4} \qquad .. (13)$$

where

$$\chi = a_0 \vec{K}_1 ; \vec{t} = a_0(\vec{K}_p - \vec{K}_q)$$

$$t_{1,2} = a_0(\vec{K}_p \mp \vec{K}_q) ; \chi_{max} = a_0\left[\frac{m}{M} K_p^2 - \frac{2m}{\hbar^2} |E_b|\right]^{1/2} \qquad ...(14)$$

Equation (13) gives exactly the same expression for crosssection as that of Mott & Massey (1949) if we neglect the distortion of the ejected electron.

Conclusion

In our treatment, the differential crosssection for the process is given by equations (11) & (12).

In equation (12) a feature which deserves special mention is that the computation of Bates & Griffing and our computed result lead essentially to the same result so long as we neglect the exchange interaction. The square of the matrix element (which is proportional to the cross-section) is in powers of K_p as K_p^{-12}, however the appearance of exchange interaction as shown by expression (12) cancels this K_p^{-12} term and thus the leading term becomes higher than K_p^{-12} (the inverse power is enhanced). As energy E is proportional to K_q^2, it is proportional to E^{-6}.

Thus $d\log I/d\log E$ must be <-6. Looking back to the experimental result by Bates & Griffing, we see from the diagram (Fite *et al* 1960) that the experimental curve falls more sharply than the curve of Bates & Griffing for high values of K_p, which means $d\log I/d\log E$ is of the order of $-\mu$ where $\mu>6$. Thus the result of present calculation seems to favour the experimental results rather than those of Bates & Griffing.

The reason why we have estimated the nature of the curve rather than fully computing it, is due to difficulty in the numerical computation, which can only be done by a high speed computer. The results of such computation will be communicated elsewhere.

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